# Privatization of public firm and urban unemployment in a dualistic economy

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#### Abstract

This paper presents a model in which the economy comprises a mixed oligopoly in an urban area and an agricultural sector in a rural area. Using it, the effect of privatization of public firms in the urban area on urban unemployment are analyzed. We introduce a mixed oligopoly model into Harris and Todaro model to analyze this effect. The results of our analysis show that privatization of public firms might not improve the unemployment rate in an urban area.

## 1 Introduction

This paper presents an examination of the effect of privatization of public firm on urban unemployment rate in a dualistic economy. Recently many researchers have studied mixed oligopoly, in which a public firm aims to maximize social welfare and private firms seek their own profit maximization. Particularly, some developing countries such as China are proceeding with privatization of public firms. Moreover, urban areas face severe unemployment because agricultural workers migrate from rural areas to urban areas. Regarding urban unemployment problems, Harris and Todaro [3] elucidated a mechanism that brings about unemployment caused by migration between urban areas and rural areas in developing countries. However, they do not discuss privatization of public firms because they deal with competitive markets in urban and rural sectors. Regarding mixed oligopoly, many researchers have analyzed this issue: Matsumura [4], [2], and others. Nevertheless, few studies deal specifically with urban unemployment under the mixed oligopoly model. Therefore, we extend Harris and Todaro [?] by combining them with a mixed oligopoly model. The paper proceeds as follows. The next section outlines the basic model. Section 3 describes the effect of privatization of public firm on the equilibrium. Finally, Section 4 presents our concluding remarks.

### 2 The model

### 2.1 Households

The economy comprises mixed oligopoly in an urban area and an agricultural sector in a rural area. All households have identical preferences. We assume that the number of households in an economy is normalized to one. They reside in urban or rural areas, and they are mobile between regions. Two kinds of household reside in urban area. Although one is labor employed in the manufactured goods sector and obtains wage income; the others are out of work. They have no income except for redistribution. The residents in the rural area are engaged in the agricultural goods sector. They receive wage income  $w_y$ . We assume that the urban wage is fixed above the market-clearing level, and that it has downward rigidity attributable to the minimum wage system and other reason. Particularly, let  $w_x$  represent the minimum wage in the urban area, which is higher than the rural wage  $w_y$ . That rural wage is determined in the labor market of the agricultural goods sector. It is equal to the marginal product of labor in the agricultural goods sector. <sup>1</sup> Similar to Fujiwara [2], each household has the following utility function  $U_i$ .

$$U_i = aq - \frac{1}{2}q^2 + z, \quad (i = x, y, u)$$
 (1)

where q and z respectively denote manufactured goods consumption and agricultural goods consumption, which are numeraire. Here the budget constraint of each household is

$$w_i + I = pq + z, \quad (i = x, y, u) \tag{2}$$

where p and I respectively denote the prices of manufactured goods and redistribution for each household. Maximizing (1) subject to (2), the demand function of manufactured goods is given as shown below.

$$Q = a - p \tag{3}$$

Moreover, each household's indirect utility function is derived as follows.<sup>2</sup>

$$v_i = \frac{1}{2}(a-p) + w_i + I, (i = x, y, u)$$
(4)

Next we refer to urban unemployment. Let  $L_x$  and  $L_u$  respectively represent the number of households employed in the manufactured goods sector and unemployed households. Here we define  $\lambda$  as the unemployment rate in the urban area.

$$\lambda \equiv \frac{L_u}{L_x + L_u} \tag{5}$$

<sup>&</sup>lt;sup>1</sup>Let x, y, and u respectively represent the manufactured goods sector, agricultural goods sector, and unemployment.

<sup>&</sup>lt;sup>2</sup>Here  $w_u$  is equal to zero because they are not employed in the manufactured goods sector.

Following Harris and Todaro [?], the expected utility in an urban area is equal to the utility in the rural area in equilibrium. Substituting (4) into this condition, the migration equilibrium condition between regions is as follows.

$$(1 - \lambda)w_x = w \tag{6}$$

Because we assume that the population is normalized, the population constraint is given as shown below.

$$L_x + L_u + L_y = 1 \tag{7}$$

Combining (5) with (7), the equation (5) can be rewritten as follows.

$$L_x + (1 - \lambda)L_y = 1 - \lambda \tag{8}$$

### 2.2 Production

#### 2.2.1 Agricultural goods sector

Next we examine the producer's behavior in detail. We assume that the manufactured good sector in the urban area is a mixed duopoly, which comprises a public firm and n private firms. However, the agricultural goods market in the rural area is competitive. Although the manufactured goods sector requires only labor as input, the agricultural goods sector requires both labor and land as input. Assuming that  $\overline{S}$  is the fixed land input and that it has normalized land input to one, we set up the production function as shown below.

$$Y = (L_y)^{\sigma}(\bar{S})^{1-\sigma}, \quad \sigma \in (0,1)$$
(9)

Because the market of agricultural goods is competitive, the following wage rate in rural areas is derived because of profit maximization.

$$w = \sigma L_u^{\sigma - 1} \tag{10}$$

Here the sum of the profit in the agricultural goods sector and land rent are redistributed to each household; also, I signifies the redistribution per capita.

#### 2.2.2 Manufactured goods sector

Presuming that there is one public firm and n private firms in the manufactured goods sector, then the market-clearing condition becomes

$$Q = a - p = x_0 + \sum_{j=1}^n x_j.$$
 (11)

¿From (??), the inverse demand function of manufactured goods is obtained as

$$p = a - x_0 - \sum_{j=1}^n x_j.$$
 (12)

Assuming that public firms and private firms have the same production technology, that is  $^3$ 

$$x_i = \frac{L_x^i}{m}, \quad (i = 0, \cdots, n).$$
 (13)

Because the only input factor to produce manufactured goods is labor and the urban wage rate  $w_x$  is fixed, the cost function of the public firm and private firms in the manufactured goods sector is given as

$$C_i(x_i) = mw_x x_i, \quad (i = 0, \cdots, n).$$

$$(14)$$

Consequently, the profit functions of public firm and private firms are derived as follows.

$$\pi_{i} = \left(a - x_{0} - \sum_{j=1}^{n} x_{j}\right) x_{i} - m w_{x} x_{i}, \quad (i = 0, \cdots, n)$$
(15)

Here the social welfare function comprises consumer surplus, the profit of the public firm, and household income, as

$$\frac{1}{2}\left(x_0 + \sum_{j=1}^n x_j\right)^2 + \pi_0 + \sum_{j=1}^n \pi_j + w_x\left(L_x^0 + \sum_{j=1}^n L_x^j\right) + wL_y + I.$$
 (16)

We consider that the government owns the share of  $(1-\theta)$  of public firms, as Matsumura assumed (1998). The purpose of public firm (firm 0) is to maximize the weighted average of social welfare and its profit, which is defined as  $V(\theta)$ .

$$V(\theta) = \pi_0 + (1 - \theta) \left[ \frac{1}{2} \left( x_0 + \sum_{j=1}^n x_j \right)^2 + \sum_{j=1}^n \pi_j + w_x \left( L_x^0 + \sum_{j=1}^n L_x^j \right) + wL_y + I \right]$$
(17)

Because the private firms maximize profit and the public firm maximizes  $V(\theta)$ , the first order condition of private firms is derived as follows.<sup>4</sup>

$$a - x_0 - (1+n)x - mw_x = 0 \tag{18}$$

The public firm aims to maximize the weighted average of social welfare and its profit  $V(\theta)$  with respect to production. Consequently, the first order condition of public firm is obtained as follows.

$$a - (1+\theta)x_0 - nx - \theta m w_x = 0 \tag{19}$$

Solving (18) and (19), the Nash equilibrium outputs  $(x_0^*, x^*)$  are obtained as follows.

$$x_0^* = \frac{a + [(1-\theta)n - \theta]mw_x}{(1+n)\theta + 1}$$
(20)

$$x^* = \frac{a\theta - mw_x}{(1+n)\theta + 1} \tag{21}$$

<sup>&</sup>lt;sup>3</sup>The setting of production function follows Daitoh[1]

<sup>&</sup>lt;sup>4</sup>The private firms are symmetric.

### **3** Comparative statics

Because we derived the behavior of the manufactured goods sector, agricultural goods sector, and households in the previous section, we consider the effect of the privatization of public firm on the urban unemployment rate. Let  $X^*$  represent the total amount of manufactured goods production. Considering () and (),  $X^*$  is derived as follows.

$$Q^* = x_0^* + nx^* = \frac{(1+n\theta)a - (1+n)m\theta w_x}{(1+n)\theta + 1}$$
(22)

Consequently, differentiating (22) with respect to  $\theta$ , the following comparative statics result is obtained.

$$\frac{\partial Q^*}{\partial \theta} = -\frac{a + (1+n)mw_x}{[(1+n)\theta + 1]^2} < 0$$
(23)

From (23), we know that the privatization of public firms engenders a decrease the total output of manufactured goods in the urban area. The necessary labor input of public firm and private firm in the manufactured goods sector is given as (13), (20), and (21). Consequently, the total number of employed households in urban area  $\bar{L}_x^*$  is obtained as follows.

$$\bar{L}_{x}^{*} = \frac{m[(1+n\theta)a - (1+n)m\theta w_{x}]}{(1+n)\theta + 1}$$
(24)

Differentiating (24) with respect to  $\theta$ , we can ascertain the effect of privatization of public firm on urban employment.

$$\frac{\partial \bar{L}_x^*}{\partial \theta} = m \frac{\partial Q^*}{\partial \theta} < 0 \tag{25}$$

Consequently, privatization of public firms engenders a decrease in the required labor input, too.

Next we refer to the effect of privatization of public firm on urban unemployment. Combining (10) with (6), the unemployed labor in the agricultural goods sector satisfying the migration equilibrium condition is the following.

$$L_y = \left[\frac{(1-\lambda)w_x}{\sigma}\right]^{\frac{1}{\sigma-1}} \tag{26}$$

Substituting (24) and (26) into (8), the equilibrium unemployment rate in the urban area is determined using the following equation.

$$\bar{L}_x^* + (1-\lambda)^{\frac{\sigma}{\sigma-1}} \left(\frac{w_x}{\sigma}\right)^{\frac{1}{\sigma-1}} + \lambda - 1 = 0$$
(27)

Differentiating (27) with respect to  $\theta$ , the following relation between urban unemployment and the privatization level of the public firm is obtained.

$$\frac{\partial \lambda}{\partial \theta} = -\frac{\frac{\partial L_x^*}{\partial \theta}}{\left(\frac{\sigma}{1-\sigma}\right)(1-\lambda)^{\frac{1}{\sigma-1}}\left(\frac{w_x}{\sigma}\right)^{\frac{1}{\sigma-1}}+1} > 0$$
(28)

Summing up, we derive the following proposition.

**Proposition 1** If the government proceeds in the privatization of public firms in an urban area, then its privatization decreases the total output of manufactured goods. Moreover, the urban unemployment rate worsens because of the public firm privatization.

## 4 Concluding remarks

As described in this paper, we construct the model by combining the Harris and Todaro model with a mixed oligopoly model. This paper presents an examination of the effect of privatization of public firms on urban unemployment. The main contribution of this paper is the demonstration that the privatization of public firms engenders a decrease in the total output of manufactured goods and worsens the urban employment. The results of our analysis in this paper are that it is necessary to consider the effect of privatization of public firm on urban employment because privatization of public firms can decrease the required labor input in an urban area. These results fit well into some reality of urban unemployment and privatization of public firms in developing countries such as China.

We simplify the model by setting its analysis simply. Particularly, we assume a linear production function of the manufactured goods sector and a quasi-linear utility function in this paper. Consequently, it is necessary to analyze this issue under a more general production function or utility function.

# References

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